American University of Beirut
Math 204
Quiz I – (Fall 2017)

Time 70 minutes.

Name: ____________________ ID#: ____________________

Circle your problem solving section number below:

- Instructor: Ms Joumana Tannous

Section 1 @ 1:00 M Section 2 @ 11:00 M Section 3 @ 4:00 M

- Instructor: Mrs Maha Itani-Hatab

Section 4 @ 8:00 Tu Section 5 @ 11:00 Tu Section 6 @ 12:30 Tu

- Instructor: Ms. Michella Bou Eid

Section 7 @ 12:30 Th Section 8 @ 2:00 Th Section 9 @ 5:00 Th

- Instructor: Ms Najwa Fuleihan

Section 10 @ 8:00 Tu Section 11 @ 2:00 Tu Section 12 @ 11:00 Tu

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Solution

1- Given the matrices $A=\begin{pmatrix}1&2&3\\4&5&6\end{pmatrix}$ $B=\begin{pmatrix}1&2\\3&-1\\0&1\end{pmatrix}$, $C=\begin{pmatrix}1&2\\3&4\end{pmatrix}$ and $D=\begin{pmatrix}2&0&0\\0&3&0\\0&0&1\end{pmatrix}$, evaluate if possible:

a. $2A+B^T = 2\begin{pmatrix}1&2&3\\4&5&6\end{pmatrix}+\begin{pmatrix}1&2\\3&-1\\0&1\end{pmatrix}^T$

(3 pts) $=\begin{pmatrix}2&4&6\\8&10&12\end{pmatrix}+\begin{pmatrix}1&3&0\\2&-1&1\end{pmatrix}=\begin{pmatrix}2+1&4+3&6+0\\8+2&10-1&12+1\end{pmatrix}$

$=\begin{pmatrix}3&7&6\\10&9&13\end{pmatrix}$

b. $BA = \begin{pmatrix}1&2\\3&-1\\0&1\end{pmatrix}\begin{pmatrix}1&2&3\\4&5&6\end{pmatrix}=\begin{pmatrix}9&12&15\\-1&1&3\\4&5&6\end{pmatrix}$

(3 pts)

$ba_{11} = 1\times1+2\times4 = 1+8=9$

$ba_{21} = 3\times1+(-1)\times4) = 3-4=-1$

c. $BA+C^{-1}$

(2 pts) dim BA = 3×3

dim C^{-1} = dim $C = 2\times2$

Not defined because dim $BA \neq$ dim C^{-1}

d. $BA+6D^{-1}$

(4 pts) $|D| = 2\times3\times1 = 6$

$D_c = \begin{pmatrix}3&0&0\\0&2&0\\0&0&6\end{pmatrix}$

$6D^{-1}=\begin{pmatrix}3&0&0\\0&2&0\\0&0&6\end{pmatrix}$

$D^{-1}=\begin{pmatrix}1/2&0&0\\0&1/3&0\\0&0&1\end{pmatrix}$

9+3=12

$BA+6D^{-1}=\begin{pmatrix}9&12&15\\-1&1&3\\4&5&6\end{pmatrix}+\begin{pmatrix}3&0&0\\0&2&0\\0&0&6\end{pmatrix}=\begin{pmatrix}12&12&15\\-1&3&3\\4&5&12\end{pmatrix}$

2- Construct the (3×3) symmetric matrix $A=(a_{ij})$ where $a_{ij}=\begin{cases}2i-j & \text{for } i>j\\ i+j & \text{for } i=j\end{cases}$

(8 pts)

$$A=\begin{pmatrix} a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{pmatrix}$$

$a_{11} = 1+1=2$

$a_{21} = 2\times2-1=4-1=3$

$a_{ij}=a_{ji}$ because symmetric so $a_{12}=a_{21}=3$

$$A=\begin{pmatrix} 2 & 3 & 5\\ 3 & 4 & 4\\ 5 & 4 & 6\end{pmatrix}$$

3- Given the matrices $A=\begin{pmatrix}4 & -5\\ 3 & 1\end{pmatrix}$ and $B=\begin{pmatrix}-1 & 2\\ 0 & -3\end{pmatrix}$, determine the matrix C so that $(C-A)^{-1}=(2I+B)^{T}$

(7 pts)

$C-A=\left[(2I+B)^{T}\right]^{-1}$

$C=A+\left[(2I+B)^{T}\right]^{-1}$

$2I=\begin{pmatrix}2 & 0\\ 0 & 2\end{pmatrix}$; $2I+B=\begin{pmatrix}2 & 0\\ 0 & 2\end{pmatrix}+\begin{pmatrix}-1 & 2\\ 0 & -3\end{pmatrix}$

$=\begin{pmatrix}1 & 2\\ 0 & -1\end{pmatrix}$

$(2I+B)^{T}=\begin{pmatrix}1 & 0\\ 2 & -1\end{pmatrix}$

$\left[(2I+B)^{T}\right]^{-1}=\dfrac{1}{-1}\begin{pmatrix}-1 & 0\\ -2 & 1\end{pmatrix}=\begin{pmatrix}1 & 0\\ 2 & -1\end{pmatrix}$

$C=\begin{pmatrix}4 & -5\\ 3 & 1\end{pmatrix}+\begin{pmatrix}1 & 0\\ 2 & -1\end{pmatrix}=\begin{pmatrix}5 & -5\\ 5 & 0\end{pmatrix}$

4- Solve the matrix equation: $\begin{pmatrix} 2x+1 \\ x-y \\ 3x+5y \end{pmatrix} + \begin{pmatrix} y-2 \\ y+4 \\ 2-3y \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \\ 9 \end{pmatrix}$

(4 pts)

$$\begin{pmatrix} 2x+1+y-2 \\ x-y+y+4 \\ 3x+5y+2-3y \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \\ 9 \end{pmatrix}$$

$$\begin{pmatrix} 2x+y-1 \\ x+4 \\ 3x+2y+2 \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \\ 9 \end{pmatrix}$$

* $2x+y-1=3$ ---- (1)

* $x+4=5 \Rightarrow x=1$

* $3x+2y+2=9$ --- (2)

subst. in eq. (1)

$2+y-1=3 \Rightarrow y=2$

check eq. (2)

$3(1)+2(2)+2 \overset{?}{=} 9$

$3+4+2=9$ ✓

5- Given the matrix $A = \begin{pmatrix} 1 & 0 & x \\ 3 & y & -4 \\ 0 & 2 & 3 \end{pmatrix}$, determine the values of x and y so that

$$A^2 = \begin{pmatrix} 1 & -4 & 4x \\ 3+3y & -7 & 3x-4y-12 \\ 6 & 8 & 1 \end{pmatrix}$$

(6 pts)

$$A^2 = \begin{pmatrix} 1 & 0 & x \\ 3 & y & -4 \\ 0 & 2 & 3 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & x \\ 3 & y & -4 \\ 0 & 2 & 3 \end{pmatrix} = \begin{pmatrix} 1 & \boxed{2x} & 4x \\ 3+3y & y^2-8 & 3x-4y-12 \\ 6 & \boxed{2y+6} & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & \boxed{-4} & 4x \\ 3+3y & -7 & 3x-4y-12 \\ 6 & \boxed{8} & 1 \end{pmatrix}$$

$2x=-4 \Rightarrow x=-2$

$2y+6=8 \Rightarrow 2y=2$

$y=1$

6- Justify why each of the following statements is false.

(11 pts)

a. If B be a (2×3) matrix then B^2 is well defined

B^2 not defined because B is not a square matrix

b. If A and B are (3×3) matrices then $\left(AB^{-1}\right)^{-1}=A^{-1}B$

$$(AB^{-1})^{-1}=(B^{-1})^{-1}A^{-1}$$
$$=BA^{-1}$$

c. If A, B and C are (2×3) matrices then $\left(AB^TC^T\right)^0=I$ where I is the identity matrix

$2\times3 \quad 3\times2 \quad 3\times2$

$2 \neq 3$

Multiplication is not defined bec. no. of columns of $B^T \neq$ no. of rows of C^T

d. If A, B and C are (2×3) matrices then AB^T+C is well defined

Addition is not defined bec. $\dim(AB^T)\neq \dim C$

$2\times3 \quad 3\times2$, $3 = 3$

$2\times2 \neq 2\times3$

e. If A and B are matrices such that $AB=I$ then A is the inverse of B.

True only if A and B are square matrices.

7- Let A and D be (3×5) and (4×5) matrices, determine the sizes of the identity matrix I and the zero matrix O so that $AIO=AD^T$

(5 pts)

$3\times5 \quad 5\times5 \quad 5\times4 \qquad 3\times5 \quad 5\times4$

$5 = 5 \quad 5 = 5 \qquad 5 = 5$

$3\times4 = 3\times4$

no. of columns of A = no. of rows of $I = 5$

I is a square matrix $\Rightarrow \dim I = 5\times5$

no. of columns of I = no. of rows of $O = 5$

$\dim(AIO)=\dim(AD^T)=(3\times4) \Rightarrow \dim O = 5\times4$

8- Let $AX = B$ be a 3×3 linear system admitting a unique solution. If $x_2 = \dfrac{\begin{vmatrix} 2 & 4 & 3 \\ 0 & 0 & 3 \\ 0 & 1 & 1 \end{vmatrix}}{\begin{vmatrix} 2 & 2 & 3 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{vmatrix}}$

a. What is the original system?

(3 pts)

$$\begin{pmatrix} 2 & 2 & 3 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix}$$

b. Find x_1

(4 pts)

$$x_1 = \frac{\begin{vmatrix} 4 & 2 & 3 \\ 0 & 1 & 3 \\ 1 & 0 & 1 \end{vmatrix}\begin{matrix} 4 & 2 \\ 0 & 1 \\ 1 & 0 \end{matrix}}{\begin{vmatrix} 2 & 2 & 3 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{vmatrix}} = \frac{10-3}{2\times1\times1} = \frac{7}{2}$$

3 0 0

4 6 0

upper triangular

9- Solve for n: ${}_nP_3 - 2\,{}_{2n}C_2 + 2n = 0$

$n \geq 3$ and $2n \geq 2$, $n \geq 1$ So $n \geq 3$

(7 pts)

$$\frac{n!}{(n-3)!} - \cancel{2}\frac{(2n)!}{(2n-2)! \times \cancel{2!}} + 2n = 0$$

$$\frac{n(n-1)(n-2)\cancel{(n-3)!}}{\cancel{(n-3)!}} - \frac{2n(2n-1)\cancel{(2n-2)!}}{\cancel{(2n-2)!}} + 2n = 0$$

$$n\left[(n-1)(n-2) - 4n + 2 + 2n\right] = 0$$

$$n(n^2 - 7n + 6) = 0$$

$$n(n-1)(n-6) = 0$$

$n = 0$ rejected / $n = 1$ rejected / $n = 6$ accepted

10- Given $A = \begin{pmatrix} 3 & -1 & 6 \\ 0 & 1 & -1 \\ 1 & 0 & 2 \end{pmatrix}$, use Gaussian elimination to find A^{-1}

(6 pts)

$$\left(\begin{array}{ccc|ccc} 3 & -1 & 6 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ 1 & 0 & 2 & 0 & 0 & 1 \end{array}\right)$$

$$R_1 \leftrightarrow R_3 \left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ 3 & -1 & 6 & 1 & 0 & 0 \end{array}\right)$$

$$-3R_1+R_3 \left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 & 0 & -3 \end{array}\right)$$

$$R_2+R_3 \left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ 0 & 0 & -1 & 1 & 1 & -3 \end{array}\right)$$

$$R_3 \times (-1) \left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 & -1 & 3 \end{array}\right)$$

$$\begin{array}{l} -2R_3+R_1 \\ R_3+R_2 \end{array} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & 2 & -5 \\ 0 & 1 & 0 & -1 & 0 & 3 \\ 0 & 0 & 1 & -1 & -1 & 3 \end{array}\right)$$

$$\underbrace{\begin{matrix} 2 & 2 & -5 \\ -1 & 0 & 3 \\ -1 & -1 & 3 \end{matrix}}_{A^{-1}}$$

11- If $A^{-1} = \begin{pmatrix} 1 & 2 & 0 \\ 1 & 0 & -1 \\ -1 & 3 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} a \\ b \\ 2 \end{pmatrix}$, what should be the values of a and b so that $X = \begin{pmatrix} 6 \\ 2 \\ 3 \end{pmatrix}$ is a solution of the system $AX = B$

(4 pts)

$$X = A^{-1}B$$

$$\begin{pmatrix} 6 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 0 \\ 1 & 0 & -1 \\ -1 & 3 & 2 \end{pmatrix} \begin{pmatrix} a \\ b \\ 2 \end{pmatrix}$$

$1(a) + 2(b) + 0(2)$

$$\begin{pmatrix} 6 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} a + 2b \\ a - 2 \\ -a + 3b + 4 \end{pmatrix}$$

$$\begin{cases} a + 2b = 6 & \text{--- (1)} \\ a - 2 = 2 & \text{--- (2)} \\ -a + 3b + 4 = 3 & \text{--- (3)} \end{cases}$$

Solving (1) & (2)

$a = 4$

$4 + 2b = 6$

$b = 1$

Checking in (3)

$-4 + 3(1) + 4 = 3$

12- Given the matrices $A=\begin{pmatrix}1 & 2 & -1\\ 0 & a & 2\\ -1 & c & d\end{pmatrix}$ and $A_c=\begin{pmatrix}1 & -2 & 1\\ -2 & e & f\\ x & h & i\end{pmatrix}$ where A_c is the matrix of cofactors of A.

a. Determine the value of x

(4 pts)

$x = a'_{31}$

but $a'_{31} = (-1)^{3+1} M_{31} = M_{31} = \begin{vmatrix} 2 & -1 \\ a & 2 \end{vmatrix} = 4+a$

$\therefore \boxed{x = 4+a}$

$1 = a'_{13}$

but $a'_{13} = (-1)^{1+3} M_{13} = M_{13} = \begin{vmatrix} 0 & a \\ -1 & c \end{vmatrix} = 0 + a = a$

$\therefore \boxed{a=1}$

$a=1 \rightarrow x = 4+a = 4+1 = 5$ $\boxed{x=5}$

b. Evaluate $\det(A)$

(2 pts)

$\det(A) = 1\, a'_{11} + 2\, a'_{12} + (-1)\, a'_{13}$ select the first row

$\det(A) = 1(1) + 2(-2) - 1\,(1)$

$= 1 - 4 - 1 = -4$

c. Let $C=\begin{pmatrix} y & 0 & 0\\ 6 & 1 & 0\\ 5 & 3 & y\end{pmatrix}$, determine y so that $\det\left(AC^0B^{-1}I^TB^TC\right)=\det\left(A^{-1}\right)$

(5 pts)

$\det(AC^0B^{-1}I^TB^TC) = \det A \times \det C^0 \times \det B^{-1} \times \det I^T \times \det B^T \times \det C$

but $\det C^0 = \det I$ since C^0 is a square matrix

$\boxed{\det C^0 = 1}$ $\boxed{\det B^{-1} = \frac{1}{\det B}}$ $\boxed{\det B^T = \det B}$

$\det C = y(1)y = y^2$ C is a lower triangular matrix

$\det(AC^0B^{-1}I^TB^TC) = -4 \times 1 \times \frac{1}{\cancel{\det B}} \times 1 \times \cancel{\det B} \times \det C$

$\det A^{-1} = -4 \det C$

$\frac{1}{\det A} = -4\, y^2 \rightarrow \frac{1}{-4} = -4y^2 \rightarrow y^2 = \frac{1}{16} \rightarrow y = \pm\frac{1}{4}$

$\boxed{y = \pm\frac{1}{4}}$

13- Eleven students put their names on slips of paper inside a box. Three names are going to be taken out. How many different ways can the three names be chosen?

(2 pts)

$_{11}C_3$

14- A teacher is going to grade his 24 students on a curve. He will give 3 A's, 6 B's, 8 C's, 4 D's and 3 F's. In how many ways can he do that?

(2 pts)

$$\frac{24!}{3!\,6!\,8!\,4!\,3!}$$

15- How many different pizzas can be made with 2 choices of dough, 4 choices of topping and 3 choices of cheese?

(2 pts)

$2 \times 4 \times 3$

16- A password consists of 2 distinct letters followed by a 5-digit number from the set $\{1,2,4,5,6,7,8\}$

How many passwords can be formed if:

a. the digits are distinct

(2 pts)

26 25 7 6 5 4 3

$26 \times 25 \times 7 \times 6 \times 5 \times 4 \times 3$

b. the letter "*x*" is included and the number is even and greater than 50 000

(4 pts)

$\underset{x}{1}$ 25 4 7 7 7 4 or 25 $\underset{x}{1}$ 4 7 7 7 4

$1 \times 25 \times 4 \times 7 \times 7 \times 7 \times 4 + 25 \times 1 \times 4 \times 7 \times 7 \times 7 \times 4$

$2 \times 25 \times 4 \times 7 \times 7 \times 7 \times 4$